Enrollment No:-__________ Exam Seat No:-_______

C.U.SHAH UNIVERSITY

Summer-2015

Subject Code: 4TE02EMT2 **Subject Name: Engineering Mathematics-II**

Course Name: B.Tech Date: 18/5/2015

Semester:II Marks:70

Time:02:30 TO 05:30

Instructions:

1) Attempt all Questions in same answer book/Supplementary.
2) Use of Programmable calculator & any other electronic instrument prohibited.
3) Instructions written on main answer book are strictly to be obeyed.
4) Draw neat diagrams & figures (if necessary) at right places.
5) Assume suitable & perfect data if needed.

Q-1 Answer the following. (2 marks each) [14]

(i) Evaluate: $\int_0^{\frac{\pi}{4}} \tan^4 x \, dx$

(ii) Find the order and degree of differential equation $\left\{\frac{d^2y}{dx^2}+1\right\}^{1/2} = \left(\frac{dy}{dx}\right)^3$

(iii) Find the oblique asymptote of the curve $y^3 - x^2(6-x) = 0$

(iv) The series $\sum u_n$ of positive terms is either ___________ or ___________ but cannot be _____________.

(v) Prove that error function is an odd function.

(vi) Evaluate : $\int_0^1 \int_0^2 \int_1^2 x^2yz \, dzdydx$.

(vii) Prove that $nB(m+1,n) = mB(m,n+1)$.

Attempt any four (from Q-2 to Q-8)

Q-2 (A) Evaluate $\int_0^1 \frac{dx}{\sqrt{4x-x^2}\sqrt{4-x^2}}$ in a terms of elliptic integral. [05]

(B) Derive Reduction formula for $\int_0^{\frac{\pi}{2}} \sin^n x \, dx \quad, n \geq 2.$ [05]

(C) Evaluate $\int_0^1 \frac{x^2}{(1-x^4)^{\frac{1}{2}}} dx \cdot \int_0^1 \frac{1}{(1-x^4)^{\frac{1}{2}}} dx$ [04]

Q-3 (A) Evaluate $\iint_R x \, dx \, dy$ over the region R bounded by $y = x^2$ and $y = x + 6$ [05]

(B) Trace the curve $r^2 = a^2\cos 2\theta$. [05]

(C) Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dydx$ by changing the order of integration. [04]

Q-4 (A) Find whole length of the Lemniscate of Bernoulli $r^2 = a^2\cos 2\theta$ [05]

(B) Trace the curve $y^2(2 + x) = x^2(2 - x)$ [05]

(C) Evaluate $\int_0^\pi x\sin^7 x \cos^4 x\, dx$ [04]

Q-5 (A) Find the area bounded by the curve $r = (1 - \cos\theta)$ [05]

(B) Solve $x^2 y\, dx - (x^3 + y^3)dy = 0$ [05]

(C) Find the orthogonal trajectories of $r^n = a^n \cos n\theta$ [04]

Q-6 (A) Solve $\frac{dy}{dx} = x^3 - 2xy,\ \ y(1) = 2$ [05]

(B) Examine the convergence of the series $\frac{1}{1\cdot 2\cdot 3} + \frac{x}{4\cdot 5\cdot 6} + \frac{x^2}{7\cdot 8\cdot 9} + \cdots$ [05]

(C) Evaluate $\int_3^7 \sqrt[4]{(x-3)(7-x)}\ dx$ [04]

Q-7 (A) Evaluate $\int_0^{2a} \int_0^{\sqrt{2ax - x^2}} (x^2 + y^2)\, dxdy$ by changing into polar co − ordinates. [05]

(B) Find radius of convergence and interval of convergence of the series $\sum (-1)^n \frac{n(x+1)^n}{2}$ [05]

(C) Evaluate $\int_0^1 x^5 \sin^{-1} x\, dx$ [04]

Q-8(A) When a resistance R ohms is connected in series with an inductance L henries, an e.m.f. $10\sin wt$ volts, the current i amperes at time t and i=0 when t = 0. Show that the current at any time t is $\frac{10}{\sqrt{R^2+L^2}}\left\{\sin(t-\phi) + e^{-\frac{Rt}{L}}\sin\phi\right\}$, where $\phi = \tan^{-1}\left(\frac{L}{R}\right)$ [05]

(B) Define Leibnitz' test on alternating series and using it examine the convergence of the series $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$ [05]

(C) Define error function and show that $\operatorname{erf}(x) = \sqrt{\frac{2}{\pi}} \int_0^{x\sqrt{2}} e^{-\frac{u^2}{2}}\, du$ [04]